SELECTED PROBLEMS OF SNIPER ACOUSTIC LOCALIZATION

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Abstract

Acoustic signals of small arm's fire, the muzzle blast and the shock wave generated by a supersonic bullet in air, are difficult to mask and can be exploited for localization of the hidden sniper. The paper presents the system of acoustic measurements based on a number of both directional and omnidirectional microphones detecting the shock wave only, yielding exact solution for the sniper direction in spite of certain measurement errors in the directional measurements. The system has a self-correcting ability concerning the sound directional measurements which contributes to the system technical feasibility. Auxiliary muzzle blast measurements would yield the sniper position.

1. Introduction

Acoustic sniper localization exploits either of two acoustic phenomena: (1) the muzzle blast detected by a system of microphones, which indicates the rifle position [1], and (2) the shock wave generated by a supersonic bullet which produces a very characteristic acoustic signal [2] detected by wide-band microphones. Two directional measurements of the shock wave cone (the sound propagation direction and its arrival time at a given microphone position) suffice for evaluation of the bullet path, assuming that its velocity is constant. The time delay of the muzzle blast signal helps to pinpoint the sniper position [3].

Only the acoustic signals: the muzzle blast to some degree, and primarily the shock wave [4] generated by a supersonic bullet passing by, could be exploited for the effective sniper detection as they cannot be in principle masked. It requires the appropriate system to be installed in the protected area that makes necessary acoustic measurements and evaluates the sniper position or, at least, his direction, allowing one to direct a proper counter-fire.

Such systems are studied in recent literature [3, 5], where the two above mentioned acoustic measurements are used to localize the sniper. Assuming the known bullet velocity and correct detection of the muzzle blast among possible nearly simultaneous other blasts, these two measurements allow one to compute the sniper position with rather low accuracy; experiments show the 50% chance of successful counter-fire [6]. The substantial difficulty arises from at least two problems connected with the acoustic measurement of the muzzle blast: 1) it is the relatively low-frequency signal whose arrival time can be detected with low resolution, and 2) it propagates over rather large distance in air of generally variable properties bending the sound propagation path and somehow enlarging the propagation time. It contributes to the evaluation error of the sniper distance.

The interesting shock wave signal propagates over a rather small distance trough assumed reasonably uniform air. Moreover, it is the very characteristic N-shaped signal with the rising time much below 1μs thus the shock wave arrival time can be detected with high resolution by a proper wide band microphone measuring the high-frequency signals. It makes the acoustic measurements of the shock wave much more accurate and reliable than the measurement of the muzzle blast. Even the information about the bullet dimension - thus on the used sniper's arm - is included in the shock-wave N-shaped signal, and can be exploited for defense purposes.

These peculiar properties of the shock-wave measurements, whose significance is seemingly under- estimated in the existing literature, deserve high attention in the considered problem of the acoustic sniper localization. In fact, only the shock wave arrival time measurements are discussed in the paper. Naturally, using the shock wave measurements alone, one cannot evaluate the distance covered by the speeding bullet and the position of the sniper. The muzzle blast

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measurements must be used for that according to the well developed method presented in [5].
However, the shock-wave measurements are able to yield, as it is shown in the paper, the accurate direction of the bullet velocity, assuming its closeness to the sniper direction, that is sufficient to direct the counter-fire. Moreover, the accurate direction to sniper allows us to detect the sniper with other than acoustic observations, the optical detection of the hot muzzle gases, for instance.
The evaluation of the bullet velocity direction requires measurements of the shock wave arrival times at a number of observation points (microphones). As it is known, the shock wave generated by a supersonic bullet has the form of a cone; its axis is the bullet path (assumed straight) and the conical angle that depends on the bullet velocity (assumed constant).
The measurement data must be sufficient to solve the geometrical problem of finding the cone parameters and thus finding the sniper direction. The problem is essentially nonlinear and quite difficult if only the shock wave arrival times are known at the given microphone positions. To make easy the computation task, one can propose here the directional measurements realized by two directional microphones placed in certain distance from each other. A number of additional omnidirectional measurements would improve the localization accuracy and assure the solution uniqueness. These auxiliary microphones could be planted 'in field' over the safe - guarded area, transmitting their raw observations to the computer system by standard radio-links.

It is shown that only two directional measurements are sufficient to evaluate the cone parameters. The angular accuracy of the measurements is expected much lower than the temporal accuracy of the shock wave arrival time measurements, which can be assumed exact. A number of omnidirectional measurements (yielding exact data of the shock wave arrival times) help us to overcome the problem with inaccurate directional measurements that can contribute much to the localization accuracy.

2. The shock-wave geometry

Assuming constant velocity v and a straight bullet path, the generated weak shock wave in air has a form of a cone S_1 (Fig. 1) of axis k (the normalized vector directed against the bullet velocity, toward the sniper) and a tip O_1, the position of the bullet tip at the given observation time t_1.

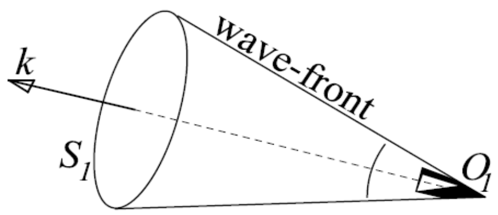

Fig. 1. The shock wave cone at the observation time t_1 when the supersonic bullet tip position is O_1.

The acoustic signal of the shock wave propagating with velocity c in the direction normal to the shock wave cone, arrives to the observers positioned at r_i (in given cartesian coordinates) at the shock wave arrival times t_i. All the observers residing on the shock wave cone S_1 would detect the wave at the same time t_1 (simultaneously). Here we only consider the shock wave generated by the bullet tip moving with the supersonic velocity $v > c$. It corresponds to the front edge of the acoustic N-shaped signal [7].
At the time $t_2 > t_1$, the shock wave cone tip moves to the point O_2:

$$\mathbf{O}_2 = \mathbf{O}_1 - v\mathbf{k}\left(t_2 - t_1\right), \tag{1}$$

and the cone broadens by the distance d normal to the cone S (Fig. 2),

$$d = c\left(t_2 - t_1\right), \tag{2}$$

where c is the sound velocity in air (constant in the assumed homogeneous air). The shock wave conical angle is

$$\sin\vartheta = \left[c\left(t_2 - t_1\right)\right] \;/\; \left[v\left(t_2 - t_1\right)\right] = c/v, \tag{3}$$

assuming the supersonic bullet.
The important conclusion results from the above that shifting the observation point r_2 (where the shock wave arrives at the time t_2) by the distance d, Eq. (2), against the outward normal n_2 to the

observed shock wave-front (the cone S_2), places the point $r_2^{'}$ on the first cone S_1 (Fig. 2); generally

$$\mathbf{r}_i^{'} = \mathbf{r}_i - c\mathbf{n}_i\left(t_i - t_1\right), \qquad (4)$$

for ith microphone measuring the shock wave arrival time t_i at position r_i.

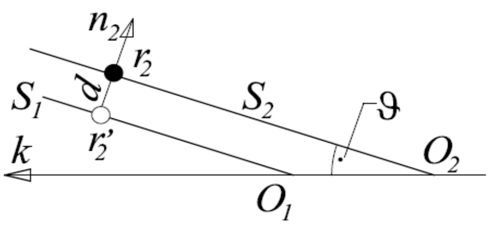

Fig. 2. The shock wave expands over time with the sound velocity in air c.

3. Directional measurements

Let the two directional microphones placed at r_1 and r_2 detecting the shock wave arrival times t_1, t_2, and simultaneously the sound propagation directions n_1 and n_2 (which are the outward normals to the shock wave cone), respectively. It is shown below that these two measurements suffice to evaluate 1) the cone axis k and 2) its tip O_1, as well as 3) the conical angle ϑ, that is 4) the bullet velocity v, and finally 5) its path in space determined by $\{O_1; k, v\}$.

Consider a line described by its point r_i and the vector n_i along it. The line is the sound ray generated at P_i and arriving at r_i at the time t_i (Fig. 3).

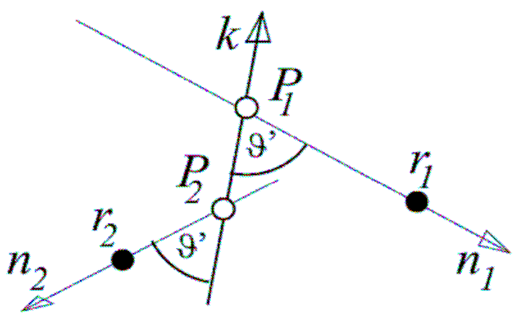

Fig. 3. The cone axis k crosses two normals to the cone surface $n_{1,2}$ at the same angle $\vartheta^{'}$.

The bullet path described by the point O_1 and the vector k crosses the rays $(r_i, n_i); i = 1; 2$, at points P_i at the same angle

$$\vartheta^{'} = \pi/2 - \vartheta, \qquad (5)$$

as it results from the geometry shown in Fig. 2. This yields the following vector equations (a dot means the scalar product)

$$\begin{aligned}&\mathbf{r}_i - \alpha_i\mathbf{n}_i = \mathbf{P}_i, i = 1, 2,\\ &\mathbf{k} = \pm\left(\mathbf{P}_1 - \mathbf{P}_2\right)/\left\|\mathbf{P}_1 - \mathbf{P}_2\right\|,\\ &\mathbf{k}\cdot\mathbf{n}_1 = \mathbf{k}\cdot\mathbf{n}_2 < 0,\end{aligned} \qquad (6)$$

which the last inequality helps us to choose the correct sign to k; α_i are unknown constants (scalars). On the strength of Eq. (2),

$$\left(\mathbf{r}_2 - \mathbf{P}_2\right)\cdot\mathbf{n}_2 - \left(\mathbf{r}_1 - \mathbf{P}_1\right)\cdot\mathbf{n}_1 = d = c\left(t_2 - t_1\right), (7)$$

and the second of Eqs. (6) multiplied by $\left\|\mathbf{P}_1 - \mathbf{P}_2\right\|$:

$$\left(\mathbf{r}_1 - \alpha_1\mathbf{n}_1 - \mathbf{r}_2 + \alpha_2\mathbf{n}_2\right)\cdot\mathbf{n}_1 = \left(\mathbf{r}_1 - \alpha_1\mathbf{n}_1 - \mathbf{r}_2 + \alpha_2\mathbf{n}_2\right)\cdot\mathbf{n}_2 , \qquad (8)$$

one can obtain:

$$\begin{aligned}&\alpha_1 = \frac{\left(\mathbf{r}_2 - \mathbf{r}_1\right)\cdot\mathbf{n}_2 + c\left(t_1 - t_2\right)}{1 - \mathbf{n}_1\cdot\mathbf{n}_2},\\ &\alpha_2 = \frac{\left(\mathbf{r}_1 - \mathbf{r}_2\right)\cdot\mathbf{n}_1 + c\left(t_2 - t_1\right)}{1 - \mathbf{n}_1\cdot\mathbf{n}_2},\\ &\sin\vartheta = -\mathbf{n}_1\cdot\mathbf{k} > 0,\quad i = 1,\ 2\ ,\end{aligned} \qquad (9)$$

where n_i; k are normalized vectors: $\left\|n_i\right\| = n_i$. n_i = 1, similarly $k\cdot k = 1$. The solution does not exist if $n_1 \cdot n_2 = 1$, which particular case (where both r_i reside on the same generatrix of the cone) is neglected in this study.

The cone tip O_1 can be evaluated from the right triangle (O_1, P_1, r_1), Fig. 4, where both ϑ and

$$\left\|\mathbf{P}_1 - \mathbf{r}_1\right\| = (\mathbf{r}_1 - \mathbf{P}_1)\cdot \mathbf{n}_1 = \alpha_1 \ , \qquad (10)$$

are known, yielding

$$\mathbf{O}_1 = \mathbf{r}_1 + \alpha_1\left(\frac{\mathbf{k}}{\mathbf{n}_1 \cdot \mathbf{k}} - \mathbf{n}_1\right) = \mathbf{P}_1 - \mathbf{k}\frac{\alpha_1}{\sin\vartheta}, \qquad (11)$$

$$v = -c/(\mathbf{n}_1 \cdot \mathbf{k}) \ .$$

This concludes the searched solution.

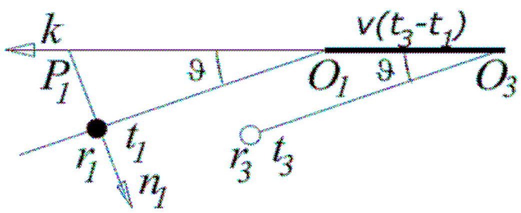

Fig. 4. The consistency condition of omni-directional measurement at r_3 results from comparison of the conical angle ϑ.

4. The measurement consistency criterion

Having three or more directional measurements of the same shock wave, the same k, v and other parameters should result from Eqs. (9), (11), evaluated from different pairs of data but including the same (r_i, n_i, t_i). For example, Eqs. (9) applied to the pairs (1; 2) and (2; 3), should yield the same α_2

$$\frac{(\mathbf{r}_1 - \mathbf{r}_2)\cdot \mathbf{n}_1 + c\ \ (t_2 - t_1)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_2} = \frac{(\mathbf{r}_2 - \mathbf{r}_3)\cdot \mathbf{n}_3 + c\ \ (t_2 - t_3)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_3}, \qquad (12)$$

in order to obtain the same P_2 and thus the same k. This consistency condition (applied for different α_i) will be exploited later in order to reduce the measurement errors causing the above equation to fail.

Now consider an omni-directional measurement of the shock wave arrival time t_3 by a microphone placed at point r_3 (Fig. 4). It is assumed here that the same shock wave (the same bullet) is detected by the directional microphones yielding the data $(r_i, n_i, t_i), i - 1, 2$, and the evaluated shock wave tip O_1 at the time t_1. The shock wave cone tip at the time t_3 is O_3:

$$\mathbf{O}_3 = \mathbf{O}_1 - v\mathbf{k}(t_3 - t_1), \qquad (13)$$

according to Eq. (1). Naturally, the cone has the same conical angle $\vartheta = \arcsin(c/v)$, hence

$$\frac{(\mathbf{r}_3 - \mathbf{O}_3)\cdot \mathbf{k}}{\left\|\mathbf{r}_3 - \mathbf{O}_3\right\|} = \cos\vartheta. \qquad (14)$$

If measurements are exact then

$$\xi = \sqrt{1 - \left[\frac{(\mathbf{r}_3 - \mathbf{O}_3)\cdot \mathbf{k}}{\left\|\mathbf{r}_3 - \mathbf{O}_3\right\|}\right]^2} + \mathbf{k}\cdot \mathbf{n}_1 \qquad (15)$$

equals zero $(\mathbf{k}\cdot\mathbf{n}_1 = -\sin\vartheta)$, otherwise $\xi \neq 0$ indicating the incorrect measurement data. The condition $\xi = 0$ is the consistency condition of omni-directional measurement data with respect to the pair of directional measurements which can include certain errors concerning the directions n_i. According to the earlier assumption that the arrival time is measured exactly, the omnidirectional measurements are considered exact.

Introducing matrix notations where vectors r_i, O_i, k, n_i are row matrices and r', k' etc. are their transposed (column) matrices, the above equation can be conveniently rewritten in the form

$$\xi^{(3)} = \sqrt{\frac{z\ \ (\mathbf{I} - k'\ \ k)\ \ z'}{z\ \ z'}} + k\ \ n'_1, \quad z = r_3 - O_3$$

. (16)

(I is a unitary matrix). All the earlier vector equations can be rewritten and evaluated in a similar manner, and the same notations are

applied to both vectors and the corresponding matrices.

The measurement consistency condition $\xi = 0$ will help us reduce the directional measurement inaccuracy. Assuming r_i, t_i known exactly and admitting certain errors δ_i in the measured shock wave propagation direction n_i, this condition yields an equation for δ_i. Several omni-directional measurements (at points r_3 ,$r_{4,....,}$) are necessary to obtain a sufficient number of equations in order to evaluate all the components of vectors δ_i of interest. Unfortunately, Eqs. (16), (9) and others are highly nonlinear and their solution may not be unique, in general. It is assumed here that the measurement can be only slightly inaccurate which allows us to apply the perturbation analysis with respect to δ_i. The resulting linear equations for the measurement errors δ_i can be easily solved. It is a matter of numerical testing how large δ_i can be admitted to obtain convergent solution for given microphone positions r_i, and the bullet miss-distance and direction $-k$.

5. Perturbation analysis

Assuming the measured (normalized) direction $n_i + \delta_i$ instead of the correct n_i, it is evident that the equation

$$\delta_i \cdot n_i = 0 \tag{17}$$

results from the normalization condition $(n_i + \delta_i) \cdot (n_i + \delta_i)$, neglecting higher order terms. Eq. (17) shows that δ_i has only two independent components orthogonal to n_i. We may choose them in directions of two orthogonal vectors:

$$\begin{aligned} e_i^{(1)} &= n_i \times r_i, \\ e_i^{(2)} &= n_i \times e_i^{(1)}, \end{aligned} \tag{18}$$

again normalized after evaluation of the vector products denoted here by x. In matrix notations:

$$\begin{aligned} e_i &= \left[e_i^{(1)}; e_i^{(2)}\right], \\ \delta_i &= d_i e_i \ , \end{aligned} \tag{19}$$

where d_i is the row matrix with two components fully characterizing the measurement errors (for the already chosen e_i).

In perturbation analysis d_i is infinitesimal, but in real computations, for δ_i small but finite, the corrected vectors $n_i \leftarrow n_i + \delta_i$ must be always normalized in order to keep the earlier equations, like Eq. (15), valid. The perturbation analysis (for infinitesimal d_i) of Eqs. (9) yields in the matrix notations and using the summation convention:

$$\begin{aligned} &\delta\alpha_i = \delta_j a_{ji}, \\ &\left[a_{ji}\right] = \frac{1}{1 - n_1 n'_2} \begin{bmatrix} \alpha_1 n'_2 & \alpha_2 n'_1 + (r_1 - r_2)' \\ \alpha_1 n'_2 + (r_2 - r_1)' & \alpha_2 n'_1 \end{bmatrix}, \end{aligned} \tag{20}$$

where $\alpha_{1,2}$ are unperturbed scalars evaluated from Eq. (9) within zero-order accuracy with respect to δ_i. To indicate the set of data: (n_1, t_1, r_1), and (n_2, t_2, r_2), used for evaluation of these coefficients, the superscript (1; 2) will be introduced in the subsequent analysis like $a_1^{(1,2)}, i = 1, 2$.

Similarly, one can obtain the perturbation equations for

$$\delta O_i = \delta_j O_{ji}, \delta k = \delta_j k_j, \ and \ \delta \xi = \delta_j \xi_j \tag{21}$$

(the unperturbed ξ has zero value). Note that again, δk is an orthogonal vector to k:

$$(\delta k) k' = 0 \tag{22}$$

because of normalization of k. It can be evaluated from the perturbation of $P_1 - P_2$, Eqs. (6); the perturbation of ξ is evaluated from Eqs. (15) provided that the perturbations to O_i, Eqs. (11) -

(13) are evaluated at first. The explicit formula are too long to be presented here; note only the introduced perturbation matrices O_{ji}, k_j, ξ_j in Eqs. (21) which will be applied in further analysis.

6. The system "2+4"

Two directional measurements introduce four unknown errors: $d_i^{(1,2)}, i=1,2$ (each δ_i has two independent components $d_i^{(1,2)}$). Four independent conditions from the measurement consistency conditions, for instance, are needed to evaluate $d_i^{(j)}, i, j=1,2$ and to retrieve the correct values of n_i. It shows that two directional and four omni-directional (introducing no extra errors) measurements of the shock wave generated by the passing by bullet are sufficient for evaluation of correct n_i in spite of the measurement errors. The correct values of n_i allow one to evaluate the correct bullet path parameters, k, O_1 and v, that is needed to obtain correct sniper localization. One can conclude that the directional measurements yield only the first guess of these parameters to ease the corresponding computation task based on the perturbation analysis.

Another measurement systems can be proposed as well. For example, three directional measurements $(n_i, t_i, i=1,2,3)$ introducing six unknowns $\left(d_j^{(1,2)}, i=1,2,3\right)$, and three omni-directional ones (at different $r_i, i=4,5,6$) which can be exploited for formulation of six consistency conditions $\xi_{4,5,6}^{(i,j)}$, evaluated using different pairs of directional data: (n_1, n_3) and (n_2, n_3), for instance. The other possibility is to formulate three consistency conditions like in Eq. (12) for α_i, appended by three ξ_i chosen to obtain the best conditioned system of equation. Yet another system uses four directional measurements and two omnidirectional ones; they will be discussed below in some details. Note however that directional measurements are much more expensive than the omnidirectional ones thus the first above mentioned system, "2+4" is preferred.

At first glance, the system of four directional measurements seems self-correcting without omnidirectional measurements. Namely, one can formulate the sufficient number of consistency conditions like Eq. (12) using different pairs of the directional measurements only. Regretfully, the rank of such system appears to be only six indicating that two another equations are necessary, namely resulting from independent omnidirectional measurements.

The system of equations resulting from two directional measurement data: $t_i, n_i+\delta_i$ at positions $r_i, i=1,2,$ and four omni-directional measurements: t_j at different $r_j, j=3,\ldots,6,$ results from four consistency criteria $\xi^{(j)}$, Eq. (16). Explicitly, according Eq. (21):

$$\delta_i \xi_i^{(j)} = x_j, i=1,2, j=3,4,5,6, \qquad (23)$$

what can be further transformed using Eq. (19) to obtain the complete system of equations for the unknown $d_j^{(l)}; i, j=1,2$:

$$\sum_{i,l=1}^{2} d_i^{(l)} q_{ij}^{(l)} = x_j, q_{ij}^{(l)} = e_i^{(l)} \xi_i^{(j)} \qquad (24)$$

where the values of x_j are ξ_j evaluated from Eq. (16) using the measurement data (that is the values of n_i including certain error δ_i what causes $\xi_j \neq 0$). This solved, yields the measurement errors δ_i which subtracted from the measured data yield the correct directions n_i. In practice, δ_i are not infinitesimal and the values of $\delta_i^{(j)}$ are evaluated from inaccurate values $\overline{n_i} = n_i + \delta_1$. Although the $n_i + \delta_1$ is considered closer to the correct n_i, it is evident that the correct solution can be obtained repeating the calculations in a recursive manner. If convergent, they yield the searched correct n_i, and finally the correct bullet path parameters, particularly the most important k.

7. Numerical example

The first considered system, "2+4," is the cheapest one. Numerical results show that happily, it performs also better than others, yielding a convergent system of equations for larger domains of bullet path parameters and larger directional measurement inaccuracy.

In the numerical example presented here for the system "2+4", the microphones are placed on the ground (this is also the sniper's post level) and distributed over the protected area about 20m long (Fig. 5; squares represent directional microphones, circles - omnidirectional ones). The bullet missdistance is assumed 2m above the ground, and its velocity is $3c$. The directional inaccuracy is modeled by performing calculations for 20 random directions $n_i + \delta_1$ within 1% limit off the correct n_i .

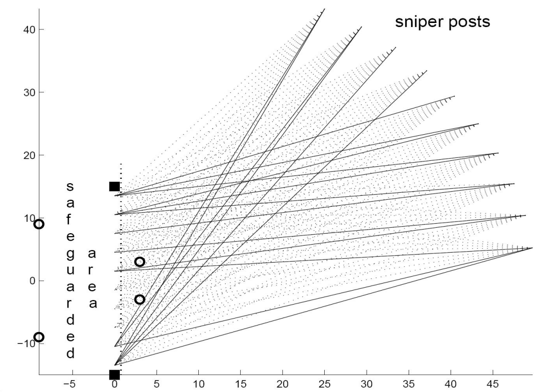

Fig. 5. The simulation of the system "2+4" for 1% measurement inaccuracies of directional measurements. Axes units are 1[m]. Dot lines shows successful evaluation of the sniper direction k , and solid lines indicate cases of not convergent iterations.

If all calculations converge to correct n_i than the corresponding bullet path is plotted with dash, otherwise with solid line. The example shows that the system fails in only few cases of sniper's fire aimed at different points from different sniper's posts.

More extensive simulations, beyond the scope of this paper, for realistic cases of the microphone distribution, sniper's positions and bullet path orientation with respect to the protected area, would reveal the true value of the above proposed systems.

Having the bullet path evaluated (characterized by O_1, k, v), one can easily exploit the other acoustic information about the fire - the acoustic signal of the muzzle blast. Assuming a small miss-distance of the sniper's fire and sufficiently large distance (L) to the sniper, the muzzle blast signal propagates nearly along the bullet path. This gives the approximation concerning the time difference between the measured shock wave (t_1) and muzzle blast (t_0) arrival times:

$$L / c - L / v = t_0 - t_1 \tag{25}$$

(t_1, t_2, ... are assumed close), from which one easily find L and thus the sniper's position measured along the evaluated bullet path.

8. Conclusions

Concluding, the acoustic system is proposed for the localization of the sniper position fully exploiting the most reliable [8] and impossible to mask information delivered by a supersonic bullet. Making two or more directional, and a number of supplemental omni-directional measurements of the shock wave signal with adequate accuracy (note that this is a 'single-event' measurement that cannot be repeated to improve it), one can evaluate the bullet path parameters (particularly the most important bullet velocity direction - k pointing at sniper's post) and the bullet velocity with improved accuracy. The proposed system has the ability to correct the directional measurement inaccuracies. The numerical examples based on perturbation linearization of highly nonlinear equations governing the geometry of the considered problem show that this self-correcting ability works well for the measurement errors as large as 1%, which can be expected technically feasible. Fully nonlinear analysis would certainly even lower this requirement admitting still larger directional measurement inaccuracy and making the presented concept of the sniper localization even more attractive for implementation, saving precious life of peace-keepers and innocent civilians from the snipers' thread.

Instantaneous evaluation of the sniper's direction (-k) is essential for this reason also that it may enable us to apply the other countermeasures. For example, properly directed optical (infrared) sensor can pick-up the cloud of hot muzzle gases detecting precisely the sniper's post. In any cases, fast response is necessary to prevent the change of the sniper's post; perhaps automatic counter-fire is necessary. Having known of such countermeasures, the sniper would try to act fast, which would surely contribute to degradation of his fire, making it less lethal in any case.

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